Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Monday 18 October 2021 – Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced
PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\mathrm{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

(a) final velocity $v = u + at$ initial velocity, time, acceleration

at $t=0$, velocity $= 4\underset{\sim}{i}$ so $u = 4\underset{\sim}{i} = \begin{pmatrix}4\\0\end{pmatrix}$

$\underset{\sim}{v} = \begin{pmatrix}4\\0\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix} \times 2$ sub $\underset{\sim}{u} = \begin{pmatrix}4\\0\end{pmatrix}$, $\underset{\sim}{a} = \begin{pmatrix}2\\-3\end{pmatrix}$, $t = 2$

$= \begin{pmatrix}4+4\\0-6\end{pmatrix} = \begin{pmatrix}8\\-6\end{pmatrix} = 8\underset{\sim}{i} - 6\underset{\sim}{j}$ use column notation to make adding simpler.

(b) At $t=0$, position vector $= \underset{\sim}{i} + \underset{\sim}{j} = \begin{pmatrix}1\\1\end{pmatrix}$

$s = ut + \frac{1}{2}at^2$

displacement

sub $\underset{\sim}{u} = \begin{pmatrix}4\\0\end{pmatrix}$, $t = 3$, $a = \begin{pmatrix}2\\-3\end{pmatrix}$

$\underset{\sim}{s} = \begin{pmatrix}4\\0\end{pmatrix} \times 3 + \frac{1}{2}\begin{pmatrix}2\\-3\end{pmatrix} \times 3^2$

$= \begin{pmatrix}12 + 9\\0 - 13.5\end{pmatrix} = \begin{pmatrix}21\\-13.5\end{pmatrix}$

Final position vector = initial position vector + displacement vector.

position vector $= \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}21\\-13.5\end{pmatrix} = \begin{pmatrix}22\\-12.5\end{pmatrix}$

$= 22\underset{\sim}{i} - 12.5\underset{\sim}{j}$

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2.

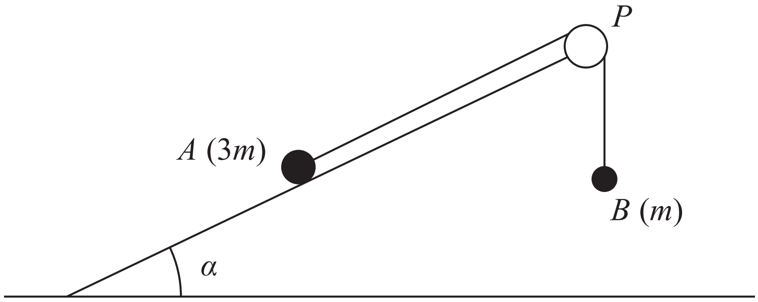


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

(c) sketch a velocity-time graph for the motion of B, from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(2)

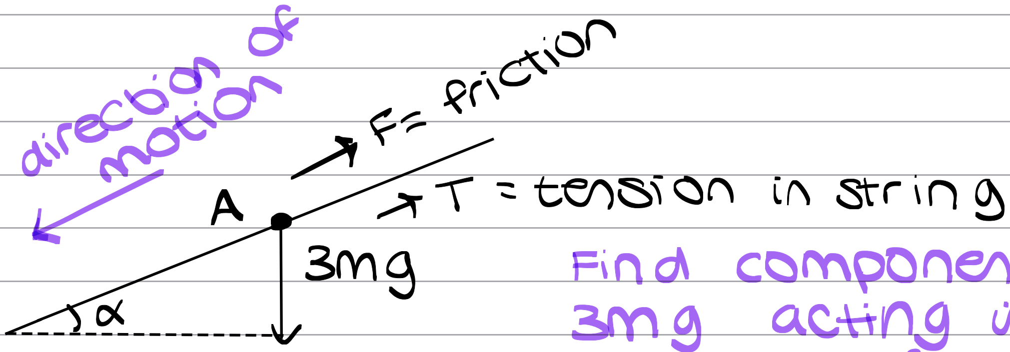
In reality, the string is not light.

(d) State how this would affect the working in part (b).

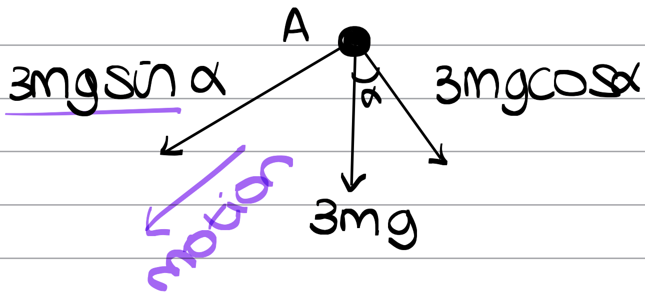
(1)

Question 2 continued

(a) consider forces acting on A



Find component of 3mg acting in direction of motion.



Resolve in components parallel and perpendicular to motion.

So 3mgsinα is component of weight of A in direction of motion.

so resultant force in direction of motion

$= 3mg\sin\alpha - F - T$

Equation of motion: $F_{resultant} = ma$

mass

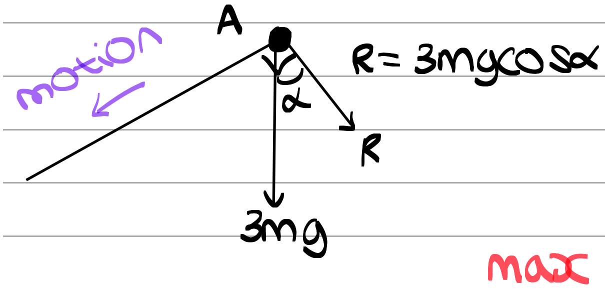
accelaration.

so, equation of motion for A:

$3mg\sin\alpha - F - T = 3ma$

Question 2 continued

(b)

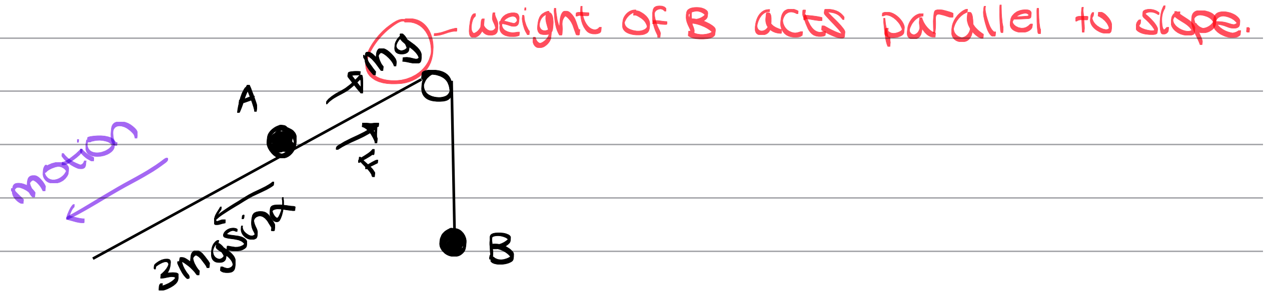


Resistance acts perpendicular to motion.

max $F = \mu R$

so $F = \frac{1}{6}(3mg\cos\alpha) = \frac{1}{2}mg\cos\alpha.$

Equation of motion for whole system:



so $F_{resultant}$ in direction of motion =

$3mg\sin\alpha - F - mg.$

so equation of motion:

$3mg\sin\alpha - F - mg = (3m + m)a.$

Substitute $F = \frac{1}{2}mg\cos\alpha$.

$3\cancel{m}g\sin\alpha - \frac{1}{2}\cancel{m}g\cos\alpha - \cancel{m}g = 4\cancel{m}a$

P 6 8 8 2 4 A 0 6 2 0

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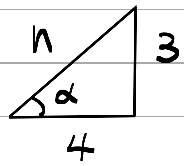
Question 2 continued

$$3g\sin\alpha - \frac{1}{2}g\cos\alpha - g = 4a.$$

$\tan\alpha = \frac{3}{4}$



By pythagoras, $h = 5$

By SOHCAHTOA, $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

sub these values in:

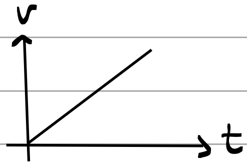
$$3g \times \frac{3}{5} - \frac{1}{2}g \times \frac{4}{5} - g = 4a$$

$$\Rightarrow \left(\frac{9}{5} - \frac{2}{5} - 1\right) g = 4a$$

$$\Rightarrow \frac{2}{5}g = 4a$$

$$\Rightarrow a = \frac{1}{10}g, \text{ as required.}$$

(c) Accelaration of B is constant, so gradient is a straight line.



(d) The string is not light so the length of each bit of string affects its weight
So the tension on A would be different to the tension on B.

(Total for Question 2 is 12 marks)

3.

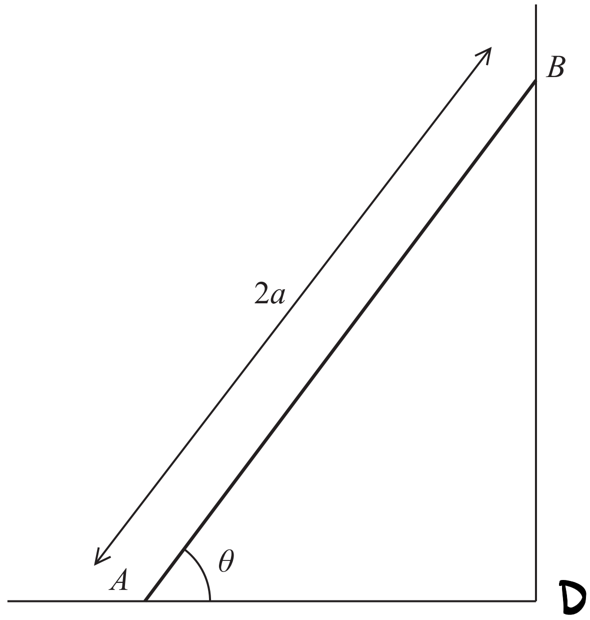


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

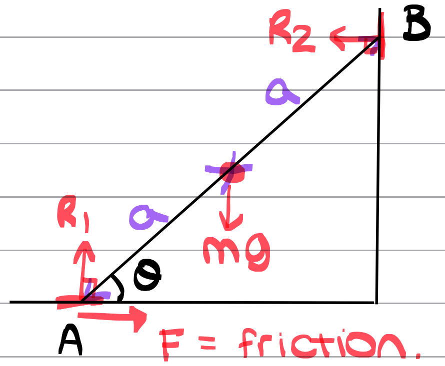
Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k.

(5)

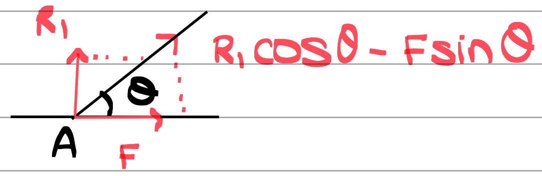
(a) moment = Force × distance (from pivot)

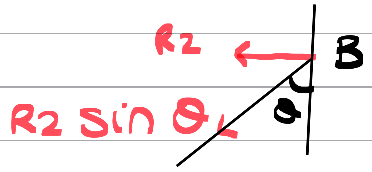
Question 3 continued



Resistance forces, R_1, R_2 act perpendicular to the plane.
Gravity (mg) acts vertically down from centre of beam.

work out components of each force that is parallel to the beam.





calculate moments = Force × distance.

moments about G:

$$R_2\sin\theta \times a = (R_1\cos\theta - F\sin\theta) \times a$$

$$\Rightarrow \quad R_2\sin\theta = R_1\cos\theta - F\sin\theta \quad (1)$$

moments about A:

moments around a pivot are equal.

$$R_2\sin\theta \times 2a = mg\cos\theta \times a.$$

$$\Rightarrow \quad R_2\sin\theta = \frac{mg\cos\theta}{2} \quad (2)$$

Question 3 continued

Now we have a pair of simultaneous equations to solve:

(1) $R_2 \sin\theta = R_1\cos\theta - F\sin\theta$

(2) $R_2 \sin\theta = \frac{mg\cos\theta}{2}$

(1) = (2) so $R_1\cos\theta - F\sin\theta = \frac{mg\cos\theta}{2}$

$\Rightarrow F\sin\theta = R_1\cos\theta - \frac{mg\cos\theta}{2}$

$F\sin\theta = \frac{(2R_1 - mg)\cos\theta}{2}$ (*)

Since the beam is in equilibrium, we can resolve it vertically

so $R_1 = mg$

substitute this into (*)

$F\sin\alpha = \frac{(2mg - mg)\cos\theta}{2}$

$\Rightarrow F = \frac{mg\cot\theta}{2}$ $\frac{\cos x}{\sin x} = \cot x$

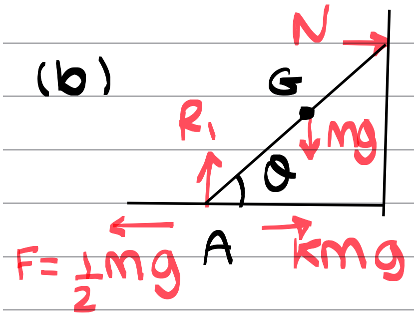
$F \leqslant \mu R$

so $\frac{mg\cot\theta}{2} \leqslant \mu \times mg$

$\Rightarrow \mu \geqslant \frac{\cot\theta}{2}$ as required.

Question 3 continued

(b)



N = Normal force, acts perpendicular to wall (force exerted on wall by beam)

At limiting equilibrium, $F = \mu R$

so $F = \frac{1}{2} \times mg$

Still in equilibrium, so $R = mg$, as above

Still in equilibrium so can resolve horizontal forces:

$$N = kmg - F$$

$$= kmg - \frac{1}{2}mg$$

consider moments about A:

force at G $\quad mg\cos\theta \times a = N\sin\theta \times 2a$ $\quad$ force at B

$$\Rightarrow \quad N\sin\theta = \frac{mg\cos\theta}{2}$$

$$N = \frac{mg\cot\theta}{2}$$

substitute $N = kmg - \frac{1}{2}mg$

(Total for Question 3 is 10 marks)

Q3(b) cont.

$$kmg - \frac{1}{2}mg = \frac{mg\cot\theta}{2}$$

$\cot x = \frac{1}{\tan x}$

$\tan\theta = \frac{5}{4}$

so $\cot\theta = \frac{4}{5}$

Rearrange + solve for k

$$\left(k - \frac{1}{2}\right)mg = \frac{1}{2}mg \times \frac{4}{5}$$

$$k - \frac{1}{2} = \frac{2}{5}$$

$$k = \frac{9}{10}.$$

4.

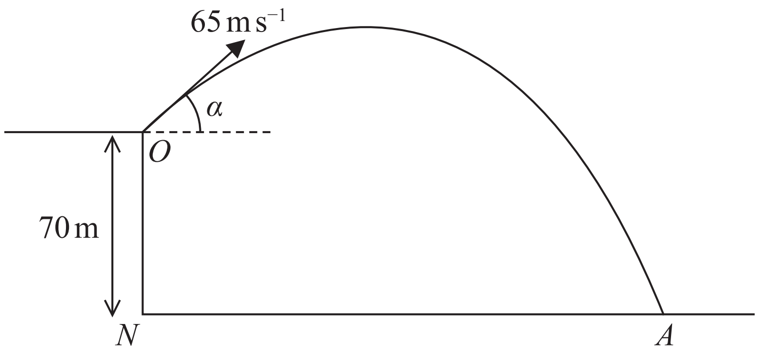


Figure 3

A small stone is projected with speed $65\,\text{m s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \dfrac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$

assumptions made.

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

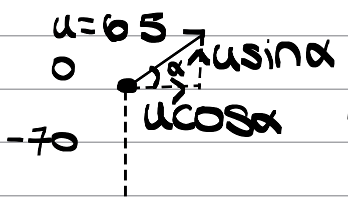
(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

(a) accelaration is constant so can use suvat equations.



↑ positive direction

↓ a=-10m/s

define which way is positive.

P 6 8 8 2 4 A 0 1 2 2 0

Question 4 continued

so, vertically we have:

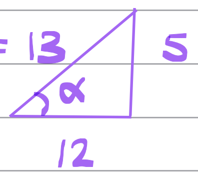
$a = -10\,m/s$, $s = -70m$, $u = 65\sin\alpha$, t is unknown

$s = ut + \frac{1}{2}at^2$

$-70 = 65\sin\alpha \times t + \frac{1}{2}(-10) \times t^2$

$-70 = 65t\sin\alpha - 5t^2$

$\tan\alpha = \frac{5}{12}$ $\sqrt{12^2 + 5^2} = 13$



$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

so $-70 = 65t \times \frac{5}{13} - 5t^2$

$5t^2 - 25t - 70 = 0$

solve with calculator or factorise or use quadratic formula.

$\Rightarrow t = 7$, $t = -2$

time cannot be negative so discount solution

so $t = 7s$

(b) we want to find speed at A.
so find final velocity.
Find horizontal and vertical components separately.

horizontally, $u_x = 65\cos\alpha$, $t = 7$, $a = 0$,
v_x is unknown

Question 4 continued

V_x, V_y : notation to separate horizontal and vertical velocity.

use $v = u + at$

$$V_x = 65\cos\alpha + 0 \times 7$$

As above, $\cos\alpha = \frac{12}{13}$

$$= 65 \times \frac{12}{13} = 60$$

vertically, $u_y = 65\sin\alpha$, $a = -10$, $t = 7$, V_y is unknown

Again, $v = u + at$

$$V_y = 65\sin\alpha + (-10) \times 7$$

$$= 65 \times \frac{5}{13} - 70$$

$$= -45$$

So $\underset{\sim}{v} = \begin{pmatrix} 60 \\ -45 \end{pmatrix}$.

Speed is the magnitude of velocity.

So speed $= \sqrt{60^2 + (-45)^2}$

$$= \sqrt{5625}$$

$$= 75 \text{ m/s.}$$

include units in answer

(c) Look at the assumptions made in the question + think about their effect:

Question 4 continued

- used an approximate value of g
- stone is not a particle so has dimensions that affect motion.
- stone may spin

etc.

By modelling as a particle, we assume it cannot spin

(Total for Question 4 is 10 marks)

5. At time t seconds, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\mathrm{m\,s^{-1}}$

(6)

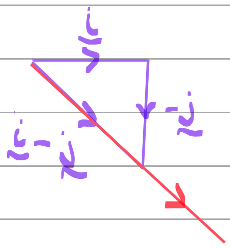
(a) Remember: $a = \frac{dv}{dt}$

so differentiate $\underset{\sim}{v}$.

$$\underset{\sim}{a} = \frac{d}{dt}\left(3t^{\frac{1}{2}}\underset{\sim}{i} - 2t\underset{\sim}{j}\right)$$

$$= \frac{3}{2}t^{-\frac{1}{2}}\underset{\sim}{i} - 2\underset{\sim}{j}$$

(b)



To travel in $\underset{\sim}{i} - \underset{\sim}{j}$ direction, have velocity parallel to $\underset{\sim}{i} - \underset{\sim}{j}$

so horizontal and vertical components must be in ratio

$\underset{\sim}{i} : \underset{\sim}{j}$

$1 : -1$

so horizontal velocity $= -$(vertical velocity)

$$\text{so} \quad 3t^{\frac{1}{2}} = -(-2t)$$

$$\Rightarrow 3\sqrt{t} = 2t$$

Question 5 continued

$$\Rightarrow \frac{3}{2} = \frac{t}{\sqrt{t}}$$

solve for t.

$$\Rightarrow \frac{3}{2} = \sqrt{t}$$

$$\Rightarrow t = \frac{9}{4} \text{ s}$$

(c) $\underline{r} = \int \underline{v} \, dt$

$$= \int 3t^{\frac{1}{2}} \underline{i} - 2t \underline{j} \; dt$$

$$= 2t^{\frac{3}{2}} \underline{i} - t^2 \underline{j} + c$$

use $\underline{r}(t=1) = -\underline{j}$ to find c.

$$\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \times 1 \\ -1 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 + c_1 \\ -1 + c_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

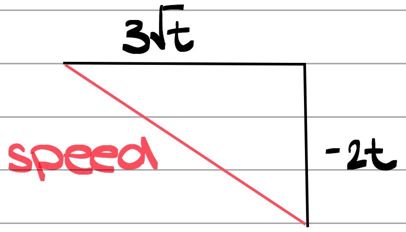
So, $\underline{r} = 2t^{\frac{3}{2}} \underline{i} - t^2 \underline{j} + -2\underline{i}$

$$= (2t^{3/2} - 2)\underline{i} - t^2 \underline{j}$$

(d) speed $= |\text{velocity}|$

Question 5 continued

so speed $= |3t^{\frac{1}{2}}\underset{\sim}{i} - 2t\underset{\sim}{j}|$



By pythagoras,

$\text{speed}^2 = (3\sqrt{t})^2 + (-2t)^2$

$\text{speed}^2 = 9t + 4t^2$

let speed = 10 to find t

$10^2 = 9t + 4t^2$

$\Rightarrow 4t^2 + 9t - 100 = 0$

$\Rightarrow t = 4, \quad t = -\frac{25}{4}$ time cannot be negative so discount this solution

so speed = 10 when t = 4.

substitute t = 4 into formula for $\underset{\sim}{r}$ to find displacement from O.

$\underset{\sim}{r} = (2 \times (4)^{\frac{3}{2}} - 2)\underset{\sim}{i} - (4)^2\underset{\sim}{j}$

$= (2 \times 8 - 2)\underset{\sim}{i} - 16\underset{\sim}{j}$

$= 14\underset{\sim}{i} - 16\underset{\sim}{j}$

Distance from O is modulus of position vector $\underset{\sim}{r}$.

$\text{distance} = \sqrt{14^2 + (-16)^2}$

Question 5 continued

$= \sqrt{196 + 256}$

$= \sqrt{452}$

$= 2\sqrt{113}$

Asks for exact value so leave as a surd

Question 5 continued

(Total for Question 5 is 14 marks)

TOTAL FOR MECHANICS IS 50 MARKS